ZWIRN
Center for Scientific Interdisciplinary
Risk and Sustainability Management

Ostfalia
University of
Applied Sciences

ZWIRN discussion papers on risk and sustainability

Predictions Between Causality and Chance

Remarks on Methods of Prediction and Limits of Predictability

Achim Michalke

Professor of Technical Management

Faculty of Supply Engineering

Ostfalia University of Applied Sciences

Wolfenbüttel, Germany

a.michalke@ostfalia.de

www.ostfalia.de/pws/michalke

ORCID-ID 0000-0001-7895-0901

April 2021

Abstract

Regularity and chance are two complementary and interacting characteristics of our world. Both are necessary to generate and maintain life. The result is a world that is predictable to a certain degree, but with clear limits as well.

Predictions are a fundamental requirement to make meaningful decisions, and we are continuously predicting how the world around us will develop. Although we know that predictions always have a limited reliability, we rarely take a closer look on precision and reliability of our predictions. Little is known about the mechanisms we use to generate predictions, about their quality, and about the limits of predictability.

This article exploits how we create predictions from the information that is available to us – recognize patterns, create a model, fill the model with data, and expand it into the future. The validity of predictions is discussed along the mechanisms that limit their precision as well as their reliability.

Keywords: *Causality, Chance, Hazard, Prediction, Predictability, Model, Probability, Error, Uncertainty, Risk, Decision, Validity, Feedback, Recursion*

Predictions Between Causality and Chance

Remarks on Methods of Prediction and Limits of Predictability

Introduction

Causality is a necessary condition for the development of life. Causality states that the future is a logical consequence of the past, thus being expectable. The development of life and its adaptation to environmental conditions is possible only if that environment behaves causally and thus predictably. This requirement is valid for our culture as well as for life itself: cultural development is possible only if we can understand our environment in causal terms, and predict consequences of our activities.

The counterpart of causality is chance. Chance states a lack of calculability, of predictability. We can understand structure and get it under control, but we cannot control chance. No surprise, we strive to understand our environment better and better, and to exclude chance where we can. Science and religion, as distant as they are to each other, both base on our desire for a well-ordered world, and for overcoming hazard. We try to avoid chance, or hazard, with the hope that either our intellect (science) or a higher authority (religion) is not at the mercy of chance.

However, chance is as indispensable for our existence as structure is. Without chance there could be no development and no free will. Without chance our fate would be determined from the beginning, without the least hope of having any influence. While we consider structure and chance as contradictory, they are in fact complementary, and both fundamental for the existence of our world. If we claim to act purposefully, we have to consider them both in their duality.

The Paradox of Predictability

Causality and chance are both characteristic for our existence

Chance has a direction in time, while structure has not. Chance constitutes the fundamental difference between past and future.

Free will and total predictability of human behaviour are incompatible with each other

The necessity of structure and chance

We discern two possible options for a future event: causal or by chance. A future event is *causal* when it originates from events in the past, and can be deducted from them by laws of development. When we know the original state as well as the laws for development, we can reliably predict the future event. The precision of our prediction is limited only by the precision of knowledge on the original state, and on the laws of development.

Alternatively, a future event may happen by *chance*, without (visible) originator in the past. At best, we can assign a probability that such a future event may happen, that is to say if that probability of chance follows certain laws, but we cannot predict its precise occurrence.

It is obvious that there is structure in our world. By and large, the future develops from the past according to laws of development. Otherwise, there would be no patterns, no possibility of directed development, nothing to think about. Natural sciences explore these laws of development and describe them as equations of motion – mathematical conditions for the development of parameters in time. Equations of motion enable us to calculate the future from the past.

> Interestingly, all equations of motion are local: Immediate future results from immediate past, in immediate vicinity of space. There is no distant effect, neither in time nor in space; there is no holistic plan that every instant has to comply with. Local equations of motion may be integrated to holistic descriptions, but with any integration there occur fundamental imprecisions which will be described later. Thus, structure may be described locally in any precision, but any global description is fundamentally fuzzy. Maybe there is a hidden relation between the locality of scientific laws and the role of chance in our world.

It is as well obvious that there is chance in our world. Chance is necessary as well, because it determines the direction of time. All equations of motion are symmetric in time: they may be applied forward and backward without difference, and they do not distinguish between directions of time. Just chance generates the difference: The past is completely determined and invariable, while the future is open. Future is always linked to hazard, future can never be completely predetermined. Chance is the prerequisite for free will, creativity, and design, chance constitutes hazard and responsibility.

Causality relies on structure, but already contains a hidden element of chance: Causality describes a deterministic chain of events, but causality also requires a direction in time. „B follows from A" is different to „A follows from B", but this difference can only be established when time has a direction, when the causal origin is invariable, but the causal effect is not.

Our existence thus necessarily requires structure *and* chance. Both factors are not contradictory, but complementary. Just their coexistence enables an inhabitable world. This is valid for our biological existence (1) as well as for our conscience. A purely structured world would be rigid, without any possibility of influence, a purely random world would be without any structure and development.

The anthropocentric approach

Metaphysics is the discipline that investigates the fundamental causality of proceedings in the world, whether our world is fundamentally causal or arbitrary. Up to about 1900, most philosophers and scientists would have agreed that the world is fundamentally deterministic and thus predictable. Quantum physics, together with some developments in mathematics (Gödel's incompleteness theorem), have skipped that conviction. Quantum physics describe the microscopic world in terms of probability, thus with an element of chance. Whether this chance is fundamental, or governed by a hidden causality, is inaccessible to us.

On macroscopic level this element of chance disappears nearly completely, explaining the validity of classic (and deterministic) physics. Elaborate equipment is necessary to discover the causal nature of quantum physics. Indirectly, however, it leaves its traces also in macroscopic events, when self-amplifying mechanisms transfer microscopic chance to the macroscopic surface, for example in heredity. In other situations, chance developments at microscopic level, lead to precisely determinable behaviour at macroscopic level, due to statistics' law of large numbers. This is valid for example in thermodynamics: while microscopic particles (molecules) behave erratic, macroscopic state variables are precisely deterministic.

Concerning our decisions we need to know, whether a prediction is de facto possible, or whether we must consider events as accidential – independent of the question whether they are in principle causal or accidental. A causal dependence is no guarantee for predictability, because we may have no access to important data, like initial conditions or laws of development.

One important reason for limited predictability are nonlinearities in the laws of motion. Under nonlinear conditions, the precision of prediction decouples from the precision that is given on initial conditions and local laws of motion. Chaotic behaviour may occur: very similar initial states lead to totally different final states. The precision that is required for initial conditions rises exponentially with the range of prediction – the span in time that shall be covered. Beyond a certain span predictions become impossible, despite causality.

There is a subtle interaction between causality and chance: chance can emerge from causality through accumulation of imprecision through integration, and causality can emerge from chance through the law of large numbers.

It may be that the do not know laws of motion, or do not know them sufficiently precise, or that we cannot determine the initial conditions precisely enough (like the geological conditions causing earthquakes or volcanic eruptions). These obstacles for prediction may be reduced by additional effort, but not completely overcome. In these cases the additional effort required rises more than proportional with the gain in predictionary precision. Prediction capabilities are then limited by economic reasons.

Wherever human beings (or more general considerately acting individuals) are involved, free will must be considered. Free will and causal prediction are incompatible: even if free will is coupled to rational considerations and emotions, it includes by definition a shot of unpredictability, called freedom. When we accept the existence of a free will, we exclude total predictability for our decisions and their consequences (see (2), in spite of esoteric exposure).

The question whether our world is in principle deterministic, or contains a fundamental ingredient of chance, is philosophically important, but without relevance for practical predictions. In practice, we are unable to precisely predict our future, leading to a de facto element of chance in future events.

> Radioactive decay of an atomic nucleus: We don‘t know whether the decay of a nucleus (time and direction) is fundamentally accidental, or follows causally from the (unknown) inner state oft he nucleus. But we know that we have in principle no way to observe the inner state in advance, among other things due to Heisenberg’s uncertainty principle. Thus, the decay is de facto accidental to us.

> Sales development of a product: Even if all participants would behave rationally, we would not be able to reliably predict the sales development of a product. The complexity of the system consisting of the enterprise itself, customers, competitors, and all other market participants is far too big to be mapped and translated into algorithms. In addition, rational behaviour does not reflect reality, and there is a free will with each single individual participating.

The mechanism of chance

In general, our environment behaves causal. We can observe regularities and derive causal rules. How enters chance this causal environment?

On the lowest microscopic level the quantum mechanical motions of law just determine probabilities. Elementary particles do not behave deterministic, but probabilistic: The probability of certain events follows strict rules and can be precisely calculated, provided we know the initial state.

> Radioactive decay of a nucleus: The decay has a certain precisely determinable probability per unit of time, but the precise time of decay and the direction in which particles spread ar not predictable. If nuclei have multiple ways of decay, each way has its own probability.

Macroscopic systems consist of a huge number of particles. These particles being atoms or molecules, they are truly identical in behaviour. Although the behaviour of each individual particle is probabilistic, the behaviour of the collective is precisely predictable, following from the law of large numbers. Probabilistic behaviour of the elements results in statistical fluctuations of macroscopic parameters, but their relative size is inversely proportional to the square root of the number of particles involved. In real cases, the size of these fluctuations is immeasurably small.

> Take atoms that have a probability of 1% to decay within one second. In a macroscopic probe there may be 10^{20} of such atoms. Within a second, an average of 10^{18} atoms will decay, with a fluctuation spread of just 10^9 atoms. That says: in 65% of all seconds, between 999,999,999x10^9 and 1,000,000001x10^9atoms will decay – a relative inaccuracy of just 10^{-9}. The biggest deviation that may have occured once since our universe exist (4x10^{17}s) is just 8.65x10^{-9}.

This mechanism among identical particles generates de facto causality from chance – but not totally. Its counterpart are nonlinear equations of motion, as for practically all real systems. Their sensitive dependence on initial conditions can self-amplify tiny fluctuations into macroscopic differences. In order to predict such systems with a growing range, we would require more and more precise knowledge of their initial state, which is at some point prevented by the probabilistic behaviour of microscopic elements, resulting in a principal limit on the range of prediction.

> Weather forecast: The development of states in the atmosphere is described (among other) by the Navier-Stokes flow equations, which are nonlinear. Today’s weather forecasts base on simulations on a lattice of air cells in area, height, and time, that inevitably generate inaccuracies by discretization. But even if we would fight discretization inaccuracies with a gigantic increase in calculation power, inaccuracies due to random events (cosmic radiation,

radioactive decay, fluctuation in chemical reactions would still remain, and limit the prediction range through nonlinear self-amplification.

Chance enters such systems by a tiny backdoor: While their laws of motion appear deterministic, they are exactly valid only for systems of unlimited size. Chance enters by way of tiny fluctuations, driven by the behaviour of microscopic particles. The effect of these fluctuations gradually grows with time and increasingly blurs predictions, until at a certain range predictions become meaningless. Even under ideal conditions, a prediction free of any randomness will be impossible.

There is no way to define the relative size of influences by causality and chance on future events, because influences by causality and chance are not comparable. Causality is a continuous influence, while chance is a discrete one. In addition, they are intertwined with each other and strengthen each other. Random events are carried into future by causal inference, thus perpetuated by causality. Vice versa, a large amount of identical random events approaches causal behaviour.

In most real cases outside natural sciences, we are quite far away from the idealistic conditions described above. Our knowledge on initial conditions is incomplete and inaccurate at best. There are no simple laws of motion, neither reductionistic nor holistic, for complex systems like those in economics or politics. We observe regularities in behaviour, but we are unable to bind them into rigid laws, because with each observance, the conditions are different. For the rules we have deduced we do not know the limits to their validity. Free will, which must be present in any system involving humans, is not accounted for in any of these rules. Still, we do predict – but what are these predictions worth?

The generation of predictions

The only source to predict the future is the past

Predictions are always imprecise

Only part of this imprecision is quantifiable

We see that it is possible to predict the future, because there is structure in our world. We also see that every prediction must be incomplete and imprecise, due to the impact of chance. In order to understand the limits of prediction we must investigate how we generate predictions (3).

Structure of a prediction

The only source of information on future events is the past. Data of the past are the only ones we can access. They must be our only basis for any prediction into future.

In a first step we try to identify *patterns* (regularities) in our observations. Regularities mean: certain combinations of events preferably occur in combination, follow each other, have proportional strength, or exclude each other. What we observe are correlations, that are combined occurences, not causalities. Whether there are causal relations between them, or whether they may occur just accidentially, is not visible to us.

We observe that it rains only when the sly is clouded, but never when the sky is clear. That is, one of the four combinations between clear / cloudy and rain / dry never occurs, namely clear plus rain. That does not prove that clouds generate rain. In fact, both events, clouds and rain, are caused by a third one, namely over-saturation of the air with water.

In the second step we generate a *model* hat expresses the observed correlations as rules. A model describes the expected development of a situation based on initial conditions, so that they fit to the observed patterns. In principle, for every set of observations one can find an infinite variety of models that are compatible with the observations. However, they are different in elements that are *not* covered by observations. As long as we don't gain access to these hidden elements, we normally select our preferred model by simplicity (Occam's razor): We favour the simplest model that is compatible with observations.

> Newton has deducted his law of gravitation from terrestrial and celestial observations. The law states that two masses attract each other proportional to their sizes and inversely proportional to the square of their distance: $F = Gm_1m_2/r^2$. This is the simplest formulation precisely compatible with the observations accessible to Newton. An alternative description is Einstein's general theory of relativity. Its formulae are too complex to present them here. However, they lead to the same consequences, except for extreme conditions like extreme mass densities (e.g., black holes) or extreme velocities (velocity of light), which were not accessible to Newton. Interestingly, both protagonists, Newton and Einstein, had to develop innovative mathematical tools for their theories.

It follows that models, even when they are well-established in their range of observation, have only limited validity in any area extending that range to unobserved terrain. Because future is in principle unobserved terrain, there follows a principal limitedness of any model being expanded into future.

A model will never cover full reality, but just a few selected parameters that we regard to be essential. Thus, we simplify reality to a level of complexity that we are able to manage. Of course, aspects of reality get lost with this act of simplification, and we assume (but don't know) that they may be irrelevant to the scope of development we intend to predict. This reduction in complexity is characteristic for every type of model, even independent of limitations in human thinking. In order to predict reality faster than it is developing, reduction in complexity is inevitable.

> Predicting rainfall, we consider parameters like water content, temperature, atmospheric pressure, solar irradiation, or velocity, but we neglect others like carbon dioxide content, electric current, or lunar phase, because we assume that they will have negligible impact on rainfall.

If possible we should *test* our model in reality. Natural sciences have developed the experiments as testing devices. A series of situations as different as possible, but describable by the model in question, are generated in a controlled environment, and true developments are compared with predictions by the model. Of particular interest are experiments that expand the range of situations to areas that have not been covered by observations yet, testing an expanded validity of the model. In particular, concurrent models can be tested against each other by generating situations where both predict different behaviours.

> In case of gravity law, Newton's and Einstein's models differ in tiny details, e.g. the perihel shift in Merkur's planetary orbit. Precise measurements on them lead to the confirmation that Einstein's model is the more accurate (though more complex) one. Areas where the models boldly differ (e.g., black holes, gravitational lenses, gravitational waves) became accessible to observation only recently.

Experiments and observations can only falsify a model, but never verify it. If observations deviate from the model's predictions, the model must be discarded, or restricted to an area where its predictions are still valid. But even the most thorough consistency of observations with the model does not confirm the model in a general way. With a deterministic model (such as most scientific laws of motion), a single counterexample is sufficient to discard the model. In contrast, a probabilistic model (that delivers just probabilities for future events) can also be falsified only with a certain level of confidence.

Finally, we will *apply* our (tested and confirmed) model by feeding it with the initial conditions of the concrete situation. Turning the model's rules on these initial conditions will provide us with a prediction of the development to be expected.

Categories of models

In an ideal case, the observation of correlations enables us to understand in detail the interactions within a system, and to describe them quantitatively. This quantitative description is called an *equation of motion*. The equation of motion typically is a description of local interaction between the parameters of the system, as a differential equation. To obtain the global development of the system in time, the equation of motion must be integrated, entering chance and the cumulative degradation of accuracy. Analytical integration, however, is possible only for very simple situations, for example gravitational systems with only two bodies. Any more complex situation requires numerical integration, entering further sources of inaccuracy, like discretization errors.

> Weather forecast is a prominent example. Equations of motion for the atmosphere (Navier-Stokes equations) describe the local relations between temperature, pressure, velocity, and other parameters. They are nonlinear, and must be integrated numerically

Formally, equations of motion may describe exact relationships. But except for very simple systems, it is practically impossible to include all parameters necessary for a full description of the system. Instead, we use summary parameters to describe relevant properties of the system, and neglect all further detail.

> in celestial motion, astronomic objects are described as point masses, at best with an inner angular momentum, and all further internal properties of these objects, like mass distributions, are neglected. Because the distances between astronomic objects are extremely large relative to their size, resulting errors are negligible.

In some cases, when a system consists of a large number of *identical* elements, it is possible to characterize it by parameters that describe holistic properties of the system (so-called state variables), integrating over the statistical behaviour of the system's elements. Equations of motion may then be defined between those state variables, totally ignoring what happens in detail on the elements' level. This is the approach of thermodynamics, as well as chemistry.

> The use of state variables requires that the system is sufficiently homogeneous, i.e. their elements being sufficiently similar, and their interactions sufficiently weak. They work well in thermodynamics, with gas molecules interacting by collisions, but they don't work in economics with enterprises interacting in most variable ways.

In case we have no access to the inner dynamics of a system (like with human decisions), or in case systems are too complex to be described by causal relationships (like with economic entities), we use the method of *extrapolation* for predictions. Extrapolation relies on the fact that systems (or system properties) in most cases do not change abruptly, but gradually. This enables us to predict the development in the near future from the development in the past by fitting a smooth development to the data of the past.

> Our observations show that clouds in the sky do not appear abruptly, but always gradually. When the sky is clear of clouds up to the horizon, we may predict that there will be little clouds during the next quarter of an hour (or more, depending on the wind), thus no rainfall will be expected.

Extrapolation does not require any knowledge of the causal mechanisms of the system, but relies on the fundamental assumption that the future is the continuation of the past. We look for a function of time that is best fitting the data of the past, and use it to calculate our prediction. However, no general procedure exists to define the “best” function fitting the data. More important than a perfect fit is that the selected function is sufficiently simple, according to Occam’s razor. Thus, we normally select an adequate class of basic functions with few parameters, then fit these parameters to the data.

> Fitting the parameters to the data can be done using the method of least squares: Each data point has a certain deviation (distance) from the final curve. Parameters are selected so that the sum of the squares of these distances is minimized. Calculation of the parameters is relatively easy, even for sophisticated functions.

The most widely used class of functions are polynomials, including the simplest ones like constant functions (degree zero) and linear functions (degree one). Extrapolating with polynomials is nothing else as a Taylor development of the (virtually) true function. In the simplest case of a constant, the value is considered independent from time, and the mean of the past is used as prediction for the future. A linear function is the extrapolation of a trend, fitted to the past by linear regression. Extrapolations of a degree two or more are rarely used, when a variation in trend shall be taken into account.

> This method is widely used for business planning. Most cost positions are considered as constants, and transferred into future without difference. More important positions, like sales, are extrapolated with a trend, either linear or exponential, like 10% increase per year. Some cost positions are coupled to sales or other lead parameters, and taken with a proportionality.

In spite of their simplicity, not in every case polynomials may be the best choice of extrapolation base. For example, when the quantity to be predicted shows a cyclic behaviour, a Fourier analysis may be the best choice. In other cases, series of exponential or other functions might be preferable. In any case, we try to select the series of functions that fits best the existing data with as few parameters as possible.

> It is not a good idea to use polynomials for predicting annual temperature developments. That does not improve when adding further degrees to the polynomial; in fact it gets worse, because the polynomial diverges just faster at some point in future. Instead, cyclic development with a base cycle of a year and a few higher-degree cycles may be the best choice.

It is obvious that an extrapolation might be a good guess for the near future, but will lose precision when the range of prediction increases, faster than a prediction based on causalities. In principle, we can increase the number of parameters (e.g. rising the degree of the polynomial), until we exactly fit every data point in the past. But that does not necessarily improve the accuracy of our prediction: Because all data have a certain error, or statistical spread, higher degrees just project these spreads into future. The possible maximum of accuracy is reached, when the fit covers all data within their spread.

In other systems events do not seem to follow deterministic rules, or these rules are inaccessible to us, and we perceive the events to occur at random. Under these circumstances only *probabilistic* predictions can be generated. In contrast to deterministic predictions, they do not predict a particular development of a quantity, a value or range for properties of future events, but give probabilities for events of a particular type to occur during a certain time.

The simplest type of a probabilistic prediction is just a probability for a certain event. In more complex cases, they may give probabilities depending on certain properties of the future event, like its strength, or a probability dependent on other future events or developments, or a probability dependent on time, in form of a probability distribution.

> Predictions on earthquakes according to the Gutenberg-Richter relation cannot predict a certain point in time when an earthquake may occur, but they give a probability that an earthquake of given strength will occur during a period of, say, one year. The Gutenberg-Richter relation couples the probability to the strength of the earthquake as a probability distribution, the probability decreasing with increasing strength of the earthquake. Probabilistic predictions of similar type will be applied to events as diverse as avalanches, forest fires, crime, changes in stock prices, blockbuster movies, or shooting stars.

> A probabilistic prediction with a cyclic time dependence concerns the occurrence of sunspots. Coverage of the sun surface with sunspots varies between 0 and 0.4% within an eleven-year cycle. Beyond that cycle, the occurrence of sunspots are random events.

Probabilistic predictions rely on data from the past, as all predictions. In the simplest case, the frequency of past events gives the probability of the future ones – again, it is assumed that the future is the continuation of the past. In more complex cases, trends, like a decrease in frequency, or other patterns. like cycles, may be taken into account.

The more reference events in the past are given, the more accurate the prediction will be, however not linearly: The statistical spread is proportional to the square root of the number of events in the past. In particular, predictions that base on only a few events in the past are highly inaccurate. Prediction that base on just a single event in the past, or predictions on events that have never occurred, are meaningless.

The stochastic character of predictions

Every prediction includes uncertainties. Depending on the type of prediction, they may occur in different shape. Strictly speaking, every prediction is a statement of probability. In predictions of deterministic type, the probability is hidden, like an error bandwidth, while probabilistic predictions openly pronounce it.

deterministic predictions describe the expected development, or target value, of a parameter into future. Uncertainty shows up as error bandwidth of future values. This error bandwidth is growing the further the prediction is expanded into the future, until it is as big as the predicted value itself, defining the limit of predictable future. Strictly speaking, the deterministic prediction states a probability distribution for the parameter, with the predicted value as expectation value.

> Weather prediction is a typical example of deterministic prediction. Developments in time for pressure, temperature, and rainfall are calculated forward. The weather forecast then consists of their expectation values, sometimes even stated together with an error bandwidth.

While the distribution of individual values around their expectancy value may be very different, calculations regularly operate with normal (Gaussian) distributions, applying the central limit theorem. The stated bandwidth of error then is the standard deviation, so that the probability of the predicted parameter hitting expectancy value ± error bandwidth is approximately 65%. As well as for measurements, no prediction is complete without stating its expected error bandwidth.

Notably, the calculated and pronounced error bandwidth always is a lower limit of error. No deterministic prediction can exclude unexpected events, resulting in a huge deviation from expected behaviour.

> In astronomy, some deterministic predictions have extreme accuracy. Timing the next total solar eclipse on September 3rd, 2081 is accurate by less than one second, at a prognostic

range of 60 years, with a relative error less than 5×10^{-10}. The major source of inaccuracy here stems from initial values. However, even this prediction is limited by unexpected events like the impact of a huge meteorite changing orbits.

Probabilistic predictions explicitly state the probability of a future event, either as a single probability value, or a probability distribution, depending on parameters, like the location, the time, or the size of the event. There is a smooth transition from deterministic to probabilistic predictions, when a probability distribution is predicted together with an expectancy value.

Typical probabilitstic predictions are those on earthquakes, volcanic eruptions, meteorite impacts, credit defaults, insurance cases, or other events with unknown causality. Predictions state the probability of occurrence within a certain time period, while a prediction of the exact time of the next event is not possible.

A special case of probabilistic predictions is the prediction of extreme values with little or no observations from the past. From non-observation in the past, the only possible type of prediction is an upper limit for their frequency, resulting from the relevant observation time in the past. Unless the probability distribution is known from theory, this upper limit is – contrary to intuition – independent of the size of such events (4).

Extreme market shifts are an important target of prediction. However, when during an observation period of 10 years the market did never shift for more than 10% in a day, then the only possible prediction states that any market shift larger than 10% in a day will probably occur not more often than once in every ten years. It may be logical, that a market shift of 20% in a day is much less probable than a market shift of 15% in an day, but there is no possibility of quantification.

The problem is that the empirical determination of a probability distribution is limited by the same lack of observation points. The error bandwidth of an empirical probability distribution is proportional to the square root of observation point, leading towards an unlimited error at the upper limit of observed size of event. An extrapolation beyond this limit is thus impossible. In addition, many empirical probability distributions of events by size show a scale-free behaviour. In this case, even the expectation value of event size is not predictable, because the central limit theorem must not be applied.

The validity and strength of predictions

The sources of every prediction are observations from the past. Every prediction relies on the fundamental assumption that the future is the continuation of the past. However, there is no guarantee – never.

A prediction can be verified or falsified by reality, provided its statements were of a deterministic character. But this intelligence after the fact won't help us – we'd like to know about the quality of predictions before the fact. Again, there is only data from the past that we can use, and with them we can investigate the rules on which our predictions rely. It is not necessary to investigate real predictions upon their success, rather full data from the past should be used to test any possible virtual predictions for validity.

Strictly speaking, deterministic rules can never be truly verified, because it is impossible to exclude that there may exist conditions (not yet observed) where the rules do not apply. In practice, however, this is not a big problem, because many rules (in particular, in natural and engineering sciences) are confirmed by an overwhelming amount of observations. Anyway, a single observation to the contrary is sufficient to falsify a deterministic rule. It is contrary to our experience of life that rules in physics or similar contexts may suddenly change. That does, however, not cover extrapolation of those rules into areas not yet covered by observations.

Probabilistic rules can neither be verified nor falsified, because there is always a non-zero probability for events turning out to be outside the predicted corridor of confidence. In particular when such rules base on a limited set of observations (like in microeconomics) it is possible that observed correlations are just accidential, and will not continue into the future.

One attempt to avoid the trap of accidential correlation is backtracking the rules. With backtracking, one part of past data is used to extract rules appropriate for predictions, while the other part is used to verify or falsify these predictions made on them. However, this procedure does not truly solve the problem. There are less data available to extract rules, making those rules more fragile. Rules that could be generated by using the full data cannot be rejected by backtracking, because the also fit the data that otherwise would be used for verification or falsification.

A further problem of reliability occurs when extremal events have to be included into predicted parameters (4). Depending on the type of probability distribution, extremal events may have (or not have) a significant impact on average values, and due to the lack of observed extremal events their probability distribution is by definition unknown. Things get worse when extremal events shall be predicted themselves.

The uncertainty of predictions resulting from the limited validity of the models they are based upon is fundamentally not quantifiable. All information that may contribute to quantification are already included into the model, leaving no data to assess uncertainty beyond the model. The error bandwidth quantified by the model is thus just a lower limit of uncertainties.

Feedback from predictions

Predictions on human behaviour can feed back on themselves

Feedback through predictions may prevent predictability

All previous considerations based on the assumption that the act of prediction itself has no influence on the future developments it is predicting. This is certainly true for developments in nature, like in astronomy or geology. However, in developments driven or influenced by human behaviour it may well be that actors are influenced by predictions, enabling the prediction to take influence on its own contents. In fact, that is the very intention of most predictions on human behaviour.

> This is the reason why poll predictions based on voter surveys must not be published in the immediate run-up of the elections. Elections shall be kept free from the influence of such predictions.

This feedback from the prediction on the predicted development may be positive (reinforcing) or negative (counteracting). In case of a positive feedback a *self-fulfilling prophecy* may be generated: Starting from an indifferent position, the prediction itself is the major driver for the development it predicts. This may happen in particular when the prediction expresses a hope or opportunity for gain.

> This mechanism frequently happens when stocks are hyped by interested parties, and followers drive the price with their subsequent purchases. Rising prices generate more followers, and the process gains speed, until the bubble bursts. The same pattern may cause falling stock prices, for example when big short sellers bet on the fall of certain stocks.

Negative feedback from predictions may be expected for predictions calling for a pending danger or loss, from the perspective of the recipients. In general, predictions with a negative feedback will tend to diminish or – in extreme cases – reverse the development they are predicting.

> The very purpose of predictions on climate development dependent on carbon dioxide emissions is to generate a negative feedback. They shall convince people to reduce carbon dioxide emissions, because consequences are shown to be unbearable for future generations.

All feedback from predictions works through humans understanding the predictions, and acting on them. The strength of this action, however, depends not only on the prediction, but on a variety of other factors, like the socialization of the prediction, that cannot be controlled, because they happen within complex systems. As a consequence it is impossible to quantify the strength of feedback, and thus adjust the prediction. It is even possible that socialization of the prediction may reverse its direction, leading to completely unpredictable behaviour.

> Precisely this happened when in January 2021 short sellers bet on losses with GameStop and other stocks. Instead of following this negative prognosis, a crowd of private investors hyped these stocks and drive them into a short squeeze.

A more subtle variety of feedback occurs when predictions or, more precise, predictability will have negative impact on the person or body whose behaviour is predicted. This type of situation occurs in virtually every competitive situation between humans: in military conflicts, economic competition, stock markets, or games like poker or chess. Participants will intentionally behave in a way that makes their behaviour unpredictable, in order not to grant an advance to their competitors. Even the possibility of a prediction leads to its impossibility.

> Stock markets are driven by beliefs on other participants' beliefs on other participants' beliefs… on behaviour. If one's own behaviour is predictable, other participants may use this knowledge to their own advantage and to one's own disadvantage. Thus, very participant will take care not to become transparent to others. This is a particular aspect of the Lucas critique on transparent markets (5).

Predictions and decision-making

The purpose of a prediction is to give us orientation in a decision situation. A decision is the selection of one alternative among others, under uncertainty. Predictions shall bridge the uncertainty and support the valuation of the alternatives. However, as discussed above, it will never be possible to avoid uncertainty.

Decisions lead to change, at least for all except one of the alternatives. in order to support *ex ante* valuation of the alternatives, Predictions must predict the development of the situation *under change* as induced by the respective alternative. This is the more difficult the more conditions would be changed by the alternative, because there are by definition no observation data for changed conditions – the only true source for any prediction.

For some predictions, in particular on natural phenomena, changes through decisions have only negligible impact. A prediction on phenomena like earthquakes, volcanic eruptions, or snowfall will not be impacted by people leaving the area, build shake-proof houses, or roofs withstanding the snow load. The same is valid for stock market predictions, when some small investors follow them to invest their pension money – although the situation will be different, when many investors follow the same prediction.

In contrast, many business decisions have a large impact on the situation – that's their purpose. This is in particular valid for all decisions on investments and organisational change, as well as innovative developments, that are targeted at a fundamentally changed situation. In contrast to gradual changes, there is no secured base for predictions of any kind. Business plans for radical changes like these are just guessed – there are no data to which they could be attached.

> A fundamental error made in many business plans is that the environment (customers, competitors etc.) is considered to be static, while the own enterprise changes radically. However, markets are highly interrelated systems, and in most cases they counteract attempts to change (competitors fight back). As a consequence, most business plans are far too optimistic: Their "*real case*" is in fact a "*best case*".

Conclusion

Predictions are possible in many circumstances, however not always, and they are always limited in precision as well as reliability. In most cases when we use predictions, we are not aware of their limitations, less still have an idea about their bandwidth of error, or degree of confidence. In fact, we regularly use predictions in many situations where they presumably cannot deliver any insight.

As predictions are an indispensable element of targeted action, we should become more conscious on the possibilities and limits of predictions, at least in professional circumstances. Little, if any, research has been designated to understand mechanisms of prediction, in particular in social sciences. The fundamental role that predictions play in our development calls for a more thorough understanding.

Literature

1. **Monod, Jacques.** *Le hasard et la nécessité.* Paris : Editions du Seuil, 1970. ISBN 3-492-01913-7.

2. **Bennett, John G.** *Risiko und Freiheit.* Zürich : Chalice Verlag, 2004. ISBN 3-905272-70-9.

3. **Silver, Nate.** *The Signal and the Noise.* s.l. : Penguin Random House, 2012. ISBN 978-0-141-97565-8.

4. **Taleb, Nassim Nicholas.** *The Black Swan.* New York, NY : Random House, 2010. ISBN 978-0-8129-7381-5.

5. **Robert E. Lucas, Jr.** Econometric Policy Evaluation: A Critique. *Carnegie-Rochester Conference Series on Public Policy.* 1976, Vol. 1.