Two Paradoxes of the Existence of magnetic Fields

Wolfenbüttel, Dezember – 14 – 2007

Claus W. Turtur, University of Applied Sciences of Braunschweig-Wolfenbuettel

Abstract

A thought experiment is considered in which somebody observes an electrical charge, moving with constant speed relatively to a given frame of reference, in which the observer is in rest. Let us further assume that no electric fields and no magnetic fields interact with this moving charge, so that there is no force acting on the charge. Consequently it keeps constant speed.
But the moving charge itself produces a magnetic field within the reference frame. Because the moving charge does not alter its speed and thereby its energy, it can not emanate any power. But the generated magnetic field contains energy, and we can calculate the power being emanated from the moving charge, which we find to be not constant as a function of time. The existence of this energy and the alteration of the power is a first paradoxon of the magnetic field.
If we follow the trace of a specified element of volume containing field energy, a further calculation proves, that this contents of energy decreases during time. This unexplained loss of energy as a consequence of the mere propagation in space is a second paradoxon of the magnetic field.

Preliminary remark

The paradoxes of magnetic fields presented here, show some similarity with two paradoxes of electrical charges and fields, which the author presented in [1].

Outline of the Article

The first paradoxon of the magnetic field is explained in the first three sections:
(1.) Calculation of the field strength of the magnetic field of a moving electrical charge
(2.) Propagation of the magnetic field respectively of its energy through the space
(3.) Calculation of the emitted power

The second paradoxon of the magnetic field is explained in the fourth section:
(4.) Tracing a cylindrical volume and its energy during its propagation through the space

Article body

(1.) Calculation of the field strength of the magnetic field of a moving electrical charge

Let us perform our calculations with the following example: The moving electrical charge which produces the magnetic field, shall be geometrically arranged homogeneously in a line with infinite length, orientated along of the z-axis, and the whole line is moving continuously in z-direction with constant speed, as illustrated in fig.1. The absolute value of the magnetic field strength $\left|\vec{H}\right|$ can than be found in a usual standard textbook for students, for instance as [2] or [3]. It is

$$\left|\vec{H}\right| = \frac{I}{2\pi r} \qquad (1)$$

with $I =$ electrical current and $r = \sqrt{x^2 + y^2}$.

Its energy density u can be found in the same textbooks. It is

$$u = \frac{\mu_0}{2} \cdot \left|\vec{H}\right|^2 \qquad (2)$$

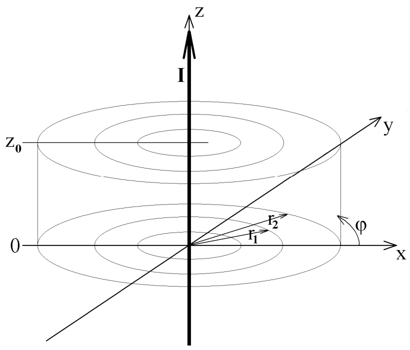

Fig. 1:
Illustration of the configuration of the moving electrical charge of our example, which produces the same magnetic field as a straight conductor with infinite length. With orientation of the conductor along the z-axis, the absolute value of the magnetic field strength can easily be given in cylinder coordinates according to equation (1).

(2.) Propagation of the magnetic field respectively of its energy through the space
The energy of the emitted magnetic field propagates with the speed of light into the xy-plane, coming radially out of the line containing the electrical charge. The reason, why we do not have to treat the propagation of the magnetic field energy as spherical, as it was done with the propagation of the electrical field in [1] is the symmetry. In [1], the electrical charge as the source of the electrical field had had spherical symmetry and the generated electrical field as well, but here the source of the magnetic field has got cylindrical symmetry and the magnetic field as well. If we look to an element of volume with the shape of a cylinder of finite length, beginning at $z=0$ and ending at $z=z_0$ (see fig.1), the energy flowing through the top and through the bottom end into the cylinder, is the same as the energy coming through those ends out the cylinder, because the cylinder is neither source nor sink. In this way, we understand, that the energy being emanated from the moving charge (which is located along the z-axis), is flowing with cylindrical symmetry into the xy-plane.

Now we want to find out, how much magnetic field energy is flowing into the space within a time interval Δt. Therefore we adjust the time-scale as following: The electrical current (i.e. the movement of the electrical charge) is switched on in the moment $t_0=0$. The time t_1 (with $t_1>0$) shall be defined as the moment, at which the magnetic field reaches the radius r_1 in consideration of its finite speed of propagation. Again a bit later, namely at the moment $t_2=t_1+\Delta t$ (with $\Delta t>0$), the field will reach a cylinder with the radius r_2. Consequently the magnetic energy, which has been emitted by the moving charge within the time interval Δt, has to be the same energy which fills the cylindrical shell from the inner radius r_1 up to the outer radius r_2. We calculate this amount of energy by integration of the energy density inside the cylindrical shell, following equation (2) in which we introduce the magnetic field according to (1):

$$W=\iiint\limits_{(\text{Zylinder})} u\,dV=\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=r_1}^{r_2}\int\limits_{z=0}^{z_0}\frac{\mu_0}{2}\cdot\left|\vec{H}\right|^2\cdot r\,dz\,dr\,d\varphi=\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=r_1}^{r_2}\int\limits_{z=0}^{z_0}\frac{\mu_0}{2}\cdot\frac{I^2}{(2\pi r)^2}\cdot r\,dz\,dr\,d\varphi$$

$$=\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=r_1}^{r_2}\int\limits_{z=0}^{z_0}\frac{\mu_0\cdot I^2}{8\pi^2 r}dz\,dr\,d\varphi=\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=r_1}^{r_2}\frac{\mu_0\cdot I^2}{8\pi^2 r}\cdot[z]_0^{z_0}\,dr\,d\varphi=\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=r_1}^{r_2}\frac{\mu_0 I^2 z_0}{8\pi^2 r}dr\,d\varphi$$

$$=\int\limits_{\varphi=0}^{2\pi}\frac{\mu_0 I^2 z_0}{8\pi^2}\cdot\left(\ln(r_2)-\ln(r_2)\right)d\varphi=\frac{\mu_0 I^2 z_0}{8\pi^2}\cdot\left(\ln(r_2)-\ln(r_2)\right)\cdot 2\pi$$

$$\Rightarrow\quad W=\frac{\mu_0 I^2 z_0}{4\pi}\cdot\ln\left(\frac{r_2}{r_1}\right)\tag{3}$$

The time interval Δt, within which this amount of energy has been emitted, can be determined from the speed of propagation of the magnetic field, which is the speed of light, as we know from the mechanism of the Hertz'ian dipole emitter [4]. It must be the same speed, with which also the electrical field propagates. Thus the time interval can be calculated as following:

$$c=\frac{r_2-r_1}{\Delta t} \quad \Rightarrow \quad \Delta t=\frac{r_2-r_1}{c} \tag{4}$$

Now we can also write r_1 and r_2 as a function of time:

$$c=\frac{r_1}{t_1} \quad \Rightarrow \quad r_1=c\cdot t_1 \tag{5}$$

and $r_2=c\cdot t_2=c\cdot\left(t_1+\Delta t\right)$ (6)

(3.) Calculation of the emitted power

From (3) we know the energy being emitted during the time interval Δt. If we calculate the quotient of this energy and the time interval Δt according to (4), we come to the emitted power as following:

$$P=\frac{W}{\Delta t}=\frac{\mu_0 I^2 z_0}{4\pi\cdot\Delta t}\cdot\ln\left(\frac{r_2}{r_1}\right). \tag{7}$$

In order to find out, whether the power P is constant in time (as it should be expected, because the charge keeps constant speed), we have to express the radii r_1 and r_2 as a function of time. For this purpose we can use the equation (5) and (6) and (7):

$$P=\frac{W}{\Delta t}=\frac{\mu_0 I^2 z_0}{4\pi\cdot\Delta t}\cdot\ln\left(\frac{c\cdot\left(t_1+\Delta t\right)}{c\cdot t_1}\right)=\frac{\mu_0 I^2 z_0}{4\pi}\cdot\frac{1}{\Delta t}\cdot\ln\left(1+\frac{\Delta t}{t_1}\right) \tag{8}$$

Obviously this expression is not constant in time. If it would be constant in time, it could not depend on the time t_1, because it should be always the same for a given Δt, independent on the moment of observation t_1. This explains the first paradoxon of the magnetic field as mentioned above: The moving charge produces a magnetic field and with it, it emanates power, but the charge itself does not alter its own energy at all, because it keeps constant speed. Paradox is the fact, that we can not see the origin of the energy emitted by the moving charge. And the first part of the paradoxon of the magnetic field has a further aspect: The emitted power is not constant in time, but we do not have any indication for an alteration of the emitted field strength.

Let us now come to the second paradoxon:

(4.) Tracing a cylindrical volume and its energy during its propagation through the space

Now we put the question, whether the energy inside the observed cylinder remains constant during its propagation into the space. Therefore we follow the observed cylinder with the inner radius r_1 and the outer radius r_2 for a further time interval $\Delta t_x>0$. Within this time, the inner radius will be enlarged until it reaches $r_3=r_1+c\cdot\Delta t_x$ and the outer radius will be enlarged until it reaches $r_4=r_2+c\cdot\Delta t_x$. So we see the following development of the situation during time:

- At the moment $t_2=t_1+\Delta t$ our cylinder had had the inner radius r_1 and the outer radius r_2, this means that we look to the same cylindrical shell as in the sections 1…3.
- At the moment $t_2+\Delta t_x=t_1+\Delta t+\Delta t_x$ this cylindrical shell from r_1 to r_2 has propagated radially into the space until it reaches the inner radius of r_3 and the outer radius of r_4.

The energy of this cylinder can be calculated in both moments according to equation (3):

- At $t_2=t_1+\Delta t$ it contains the energy $W_{12}=\frac{\mu_0 I^2 z_0}{4\pi}\cdot\ln\left(\frac{r_2}{r_1}\right)$.
- At $t_2+\Delta t_x=t_1+\Delta t+\Delta t_x$ it contains the energy

$$W_{34}=\frac{\mu_0 I^2 z_0}{4\pi}\cdot\ln\left(\frac{r_4}{r_3}\right)=\frac{\mu_0 I^2 z_0}{4\pi}\cdot\ln\left(\frac{r_2+c\cdot\Delta t_x}{r_1+c\cdot\Delta t_x}\right).$$

If we want to understand, what happens during the time interval Δt_x, we have to put r_1 and r_2 from equations (5) and (6) into W_{12} and W_{34}:

$$\text{At } t_2 = t_1 + \Delta t \Rightarrow W_{12} = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{r_2}{r_1}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{c \cdot (t_1 + \Delta t)}{c \cdot t_1}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{t_1 + \Delta t}{t_1}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(1 + \frac{\Delta t}{t_1}\right) \quad (9)$$

$$\text{At } t_2 + \Delta t_x = t_1 + \Delta t_x + \Delta t \Rightarrow W_{34} = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{r_4}{r_3}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{r_2 + c \cdot \Delta t_x}{r_1 + c \cdot \Delta t_x}\right)$$

$$\Rightarrow W_{34} = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{c \cdot (t_1 + \Delta t) + c \cdot \Delta t_x}{c \cdot t_1 + c \cdot \Delta t_x}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(\frac{t_1 + \Delta t + \Delta t_x}{t_1 + \Delta t_x}\right) = \frac{\mu_0 I^2 z_0}{4\pi} \cdot \ln\left(1 + \frac{\Delta t}{t_1 + \Delta t_x}\right) \quad (10)$$

It is clear that both expressions (9) and (10) are different. We calculated the energy inside a moving cylinder. Obviously, from $\Delta t_x > 0$ we can conclude the relation $1 + \frac{\Delta t}{t_1} > 1 + \frac{\Delta t}{t_1 + \Delta t_x}$

$\Rightarrow \ln\left(1 + \frac{\Delta t}{t_1}\right) > \ln\left(1 + \frac{\Delta t}{t_1 + \Delta t_x}\right)$. This means, that we found $W_{12} > W_{34}$.

So, our volume element had lost energy just by propagating into the space. This unexplained loss of energy can be understood as a second paradoxon of the magnetic field. We remember that our magnetic field is static (in the given frame of reference), this means that at every position which the field already reached, the field strength will remain constant during time. And the speed with which the field producing charges moves is also constant during time, beginning with the moment t_0, at which the electric current had been switched on.

The open question, which comes from both of these magnetic paradoxes is the following: From where does the energy of the magnetic field come, which is emanated by the moving charge, and where does the field energy go, when the field emanates into the space ?

Literature

[1] Two Paradoxes of the Existence of electric Charge
Claus W. Turtur, arXiv:physics/0710.3253v1 (Okt.2007)

[2] Klassische Elektrodynamik
John David Jackson, Walter de Gruyter Verlag, 1981, ISBN 3-11-007415-X

[3] Physik
Douglas C. Giancoli, Pearson Studium (2006), ISBN-13: 978-3-8273-7157-7

[4] Bergmann Schaefer, Lehrbuch der Experimentalphysik, Vol.2
Heinrich Gobrecht et.al., Walter de Gruyter Verlag, 1971, ISBN 3-11-002090-0

Adress of the Author:

Prof. Dr. Claus W. Turtur
University of Applied Sciences of Braunschweig-Wolfenbuettel
Salzdahlumer Strasse 46 / 48
Germany – 38302 Wolfenbuettel
Email: c-w.turtur@fh-wolfenbuettel.de
Internet: http://public.rz.fh-wolfenbuettel.de/%7ETurtur/physik/