

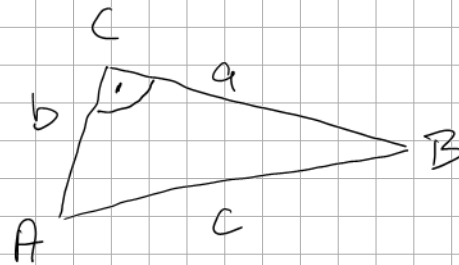
Vorkurs 1.3.2024

6. Trigonometrie

6.1 Trigonometrische Berechnungen

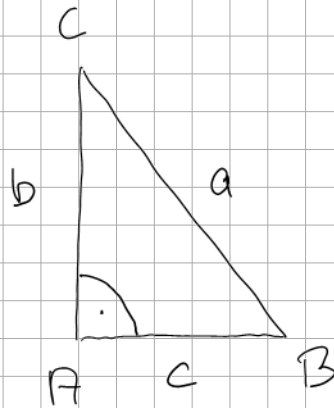
Rechtwinklige Dreiecke

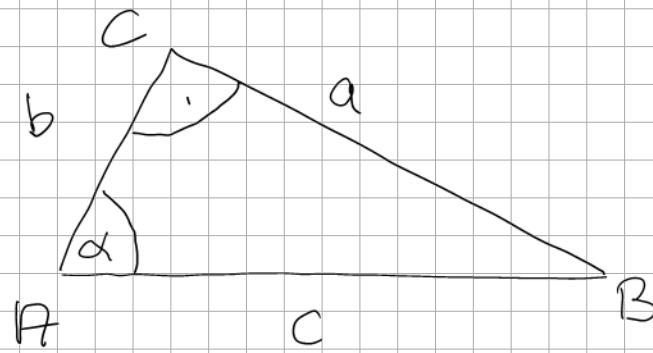
Pythagoras $a^2 + b^2 = c^2$



„ 1. Kathete² + 2. Kathete² = Hypotenuse² “

$$b^2 + c^2 = a^2$$





$$\sin(\alpha) = \frac{a}{c}$$

Gegentkathete
Hypotenuse

$$\cos(\alpha) = \frac{b}{c}$$

Ankathete
Hypotenuse

$$\tan(\alpha) = \frac{a}{b}$$

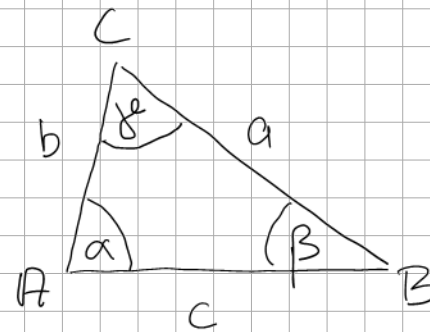
Gegentkathete
Ankathete

$$\cot(\alpha) = \frac{b}{a}$$

Ankathete
Gegentkathete

Allgemeines Dreieck

Sinussatz
$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$



Cosinussatz

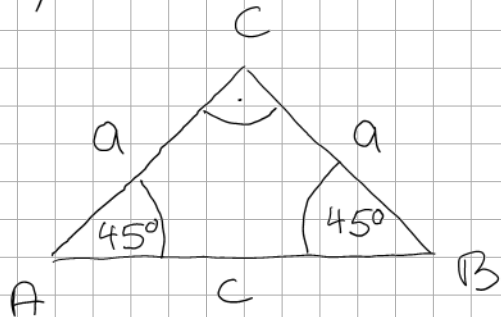
$$a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac \cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

Beispiel:

1) $\sin(\alpha)$, $\cos(\alpha)$, $\tan(\alpha)$, $\cot(\alpha)$ für $\alpha = 45^\circ$



$$c^2 = a^2 + a^2$$

$$c^2 = 2a^2$$

$$c = a\sqrt{2}$$

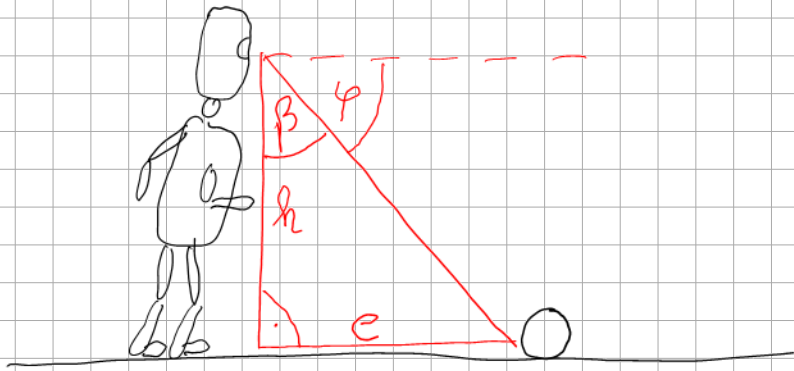
$$\sin(\alpha) = \frac{a}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\cos(\alpha) = \frac{a}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\tan(\alpha) = \frac{a}{a} = 1$$

$$\cot(\alpha) = \frac{a}{a} = 1$$

2) Robotersteuerung (Robo - Cup)



Bughöhe 28cm

$$h = 28\text{cm}$$

Neigungswinkel 40°

$$\varphi = 40^\circ$$

Wie weit ist der Ball entfernt?

e

$$\beta = 90^\circ - \varphi$$

$$\tan(\beta) = \frac{e}{h}$$

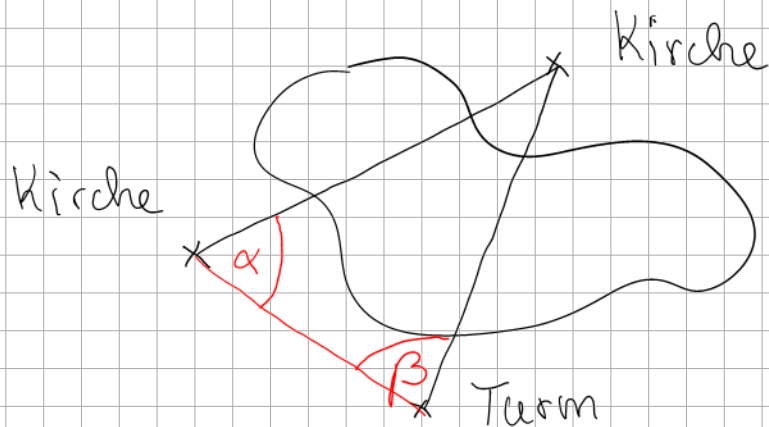
$$\beta = 50^\circ$$

$$e = h \cdot \tan(\beta)$$

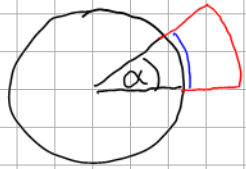
$$e = 28 \cdot \tan(50^\circ)$$

$$e \approx 33,4\text{cm}$$

3) Landvermessung



6.2 Bogenmaß



Winkel: 1) Grad 2) Bogenmaß

Bogenmaß: Bogenlänge im Verhältnis zum Radius

$\hat{=}$ Bogenlänge eines Kreises mit $r = 1$

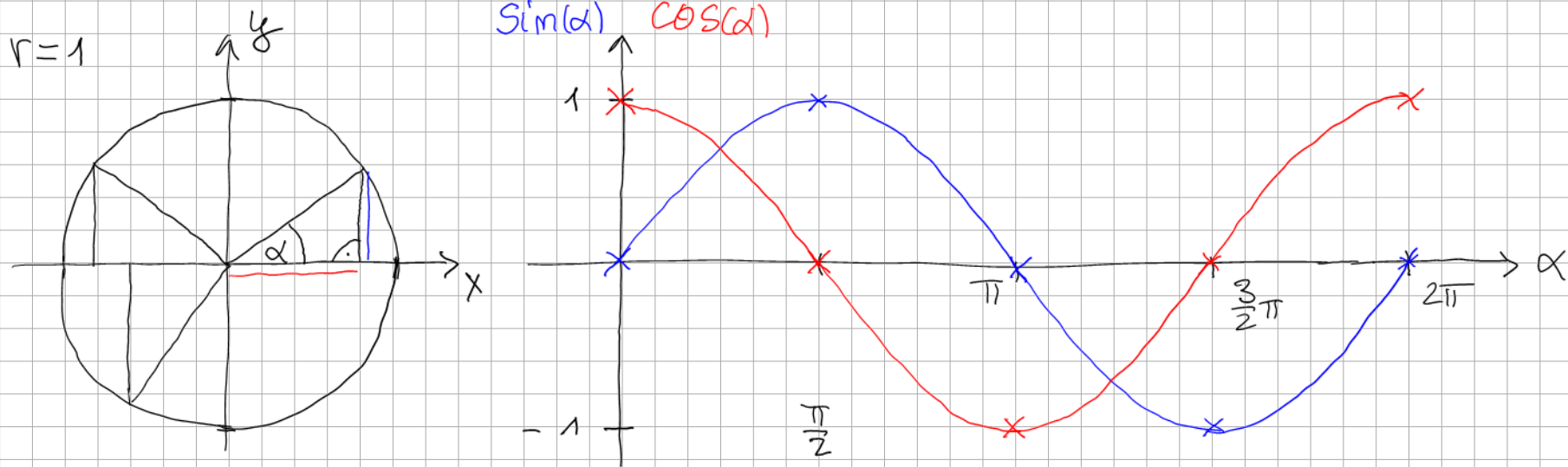
Vollwinkel $360^\circ \hat{=}$ Bogenlänge ist der Umfang des Kreises $2\pi r$

Bogenmaß $b = \frac{2\pi r}{r} = 2\pi$

| | | | | | | | | |
|----------|-------------|-------------|-----------------|------------------|-----------------|-----------------|-----------------|-------------------|
| α | 360° | 180° | 90° | 270° | 45° | 30° | 60° | 1° |
| b | 2π | π | $\frac{\pi}{2}$ | $\frac{3}{2}\pi$ | $\frac{\pi}{4}$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{180}$ |

$$b = \frac{\alpha}{360^\circ} \cdot 2\pi$$

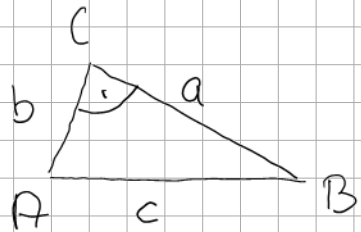
6.3 Winkelfunktionen, trigonometrische Funktionen



Periodische Funktionen $\sin(\alpha) = \sin(\alpha + k \cdot 2\pi)$, $k \in \mathbb{Z}$
 $\cos(\alpha) = \cos(\alpha + k \cdot 2\pi)$

Zusammenhang $\sin(\alpha) = \cos(\alpha - \frac{\pi}{2})$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1 \quad \sin^2(\alpha) = (\sin(\alpha))^2$$

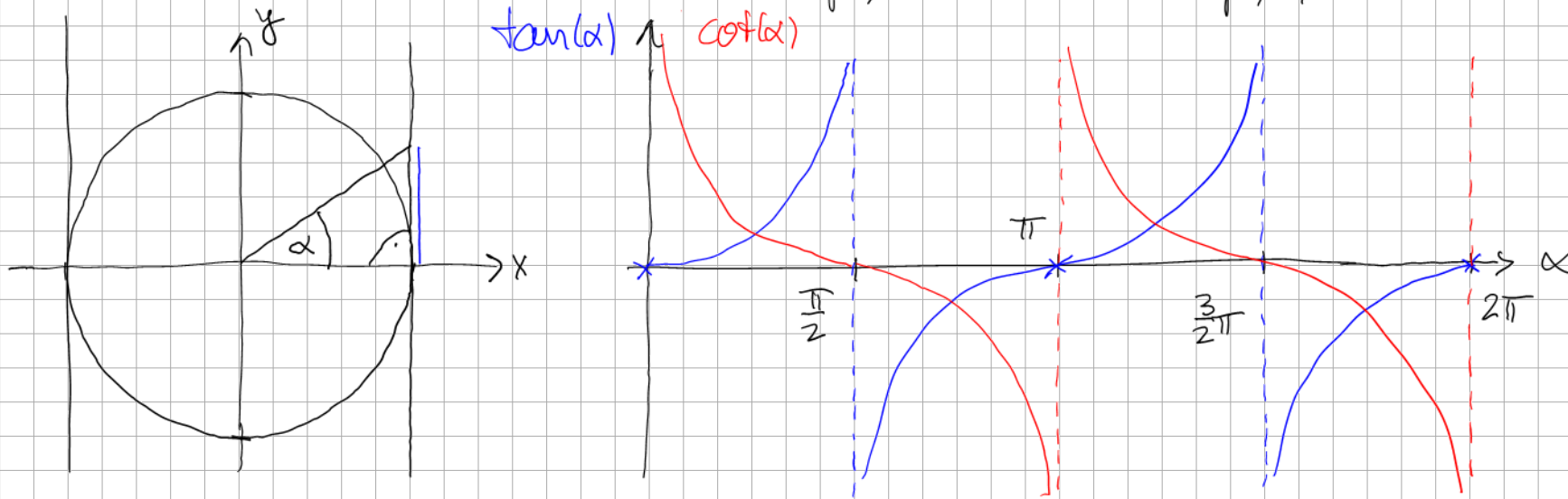


$$\sin(\alpha) = \frac{a}{c}, \quad \cos(\alpha) = \frac{b}{c}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

Additionstheoreme

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin(\alpha) \cos(\beta) \pm \sin(\beta) \cos(\alpha) \\ \cos(\alpha \pm \beta) &= \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)\end{aligned}$$



Periodische Funktionen $\tan(\alpha) = \tan(\alpha + k \cdot \pi)$, $k \in \mathbb{Z}$
 $\cot(\alpha) = \cot(\alpha + k \cdot \pi)$

Definitionslücken

$\tan(\alpha)$ ist nicht definiert für $\alpha = \frac{\pi}{2} + k \cdot \pi$, $k \in \mathbb{Z}$

$\cot(\alpha)$ ist nicht definiert für $\alpha = k \cdot \pi$

Zusammenhang zwischen Winkelfunktionen:

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

$$\cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)}$$

Aufgaben

Skript

Nr. 41 - 49

Nr. 42, 43, 44 Kantenlänge